



Ab initio **few-mode** theory

for quantum potential scattering problems

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Christoph H. Keitel and Jörg Evers

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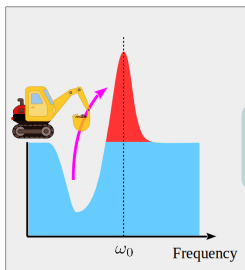
JUST TO CLEAR THINGS UP:

A FEW	ANYWHERE FROM 2 TO 5
A HANDFUL	ANYWHERE FROM 2 TO 5
SEVERAL	ANYWHERE FROM 2 TO 5
A COUPLE	2 (BUT SOMETIMES UP TO 5)

<https://xkcd.com/1070/>



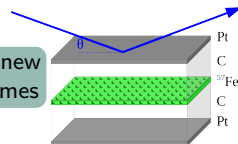
Background & Motivation



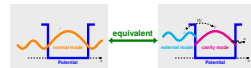
Coherent control of x-rays and nuclei

Nuclear quantum optics

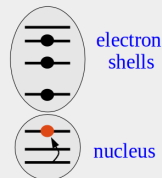
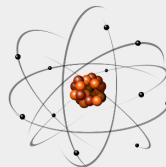
X-rays excite new quantum regimes



Understand fundamentals



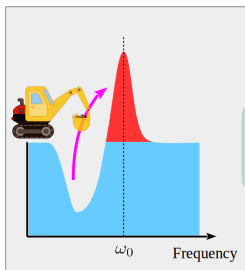
“Nuclear” quantum optics



electron shells

nucleus

Background & Motivation

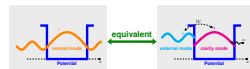
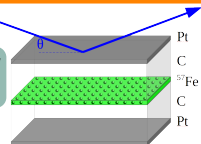


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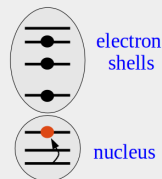
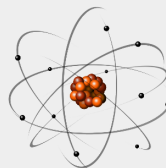


Mössbauer transitions as extreme qubits

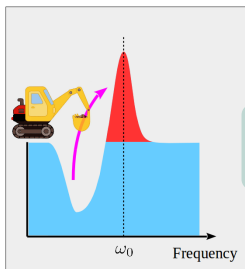
- Tiny decoherence ($Q = 10^{12}$ to 10^{24})
- Room temperature
- Solid state (no vacuum necessary)
- Large coherently interacting ensembles

Motivations: metrology, spectroscopy,...

“Nuclear” quantum optics



Background & Motivation

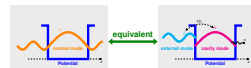
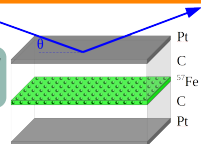


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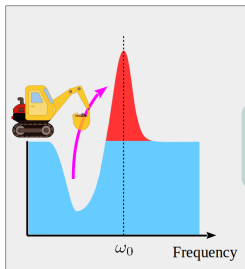
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- Röhlsberger et. al. *Science* **328** 5983 (2010)
 Röhlsberger et. al. *Nature* **482** 7384 (2012)
 Heeg & Evers *Phys. Rev. A* **88**, 043828 (2013)
 Heeg et. al. *Phys. Rev. Lett.* **111** 073601 (2013)
 Heeg et. al. *Phys. Rev. Lett.* **114** 203601 (2015)
 Heeg et al. *Phys. Rev. Lett.* **114**, 207401 (2015)
 Haber et. al. *Nature Photonics* **10** 445 (2016)
 Haber et. al. *Nature Photonics* **11** 720 (2017)

...

Background & Motivation



Coherent
control of x-rays
and nuclei

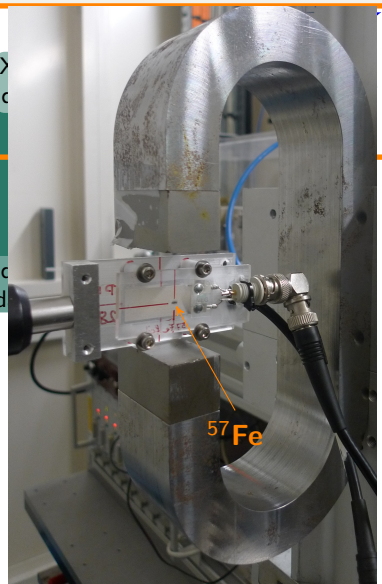
Nuclear
quantum
optics

Unc
fund

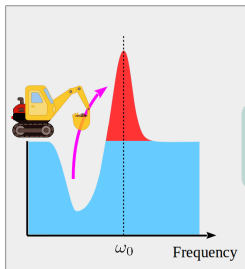
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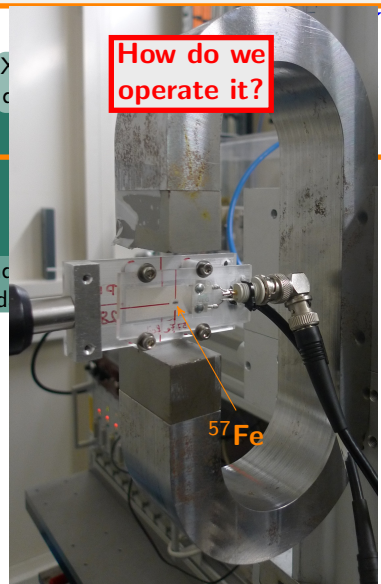
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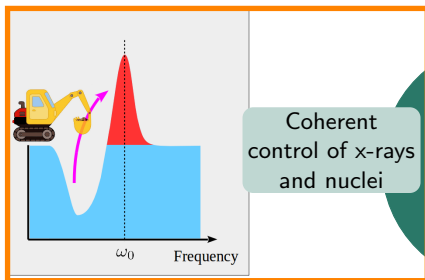
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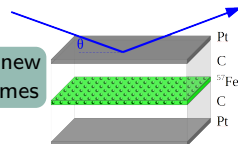


Background & Motivation

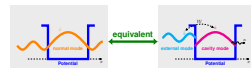


Nuclear quantum optics

X-rays excite new quantum regimes



Understand fundamentals



New X-ray sources

- 4th generation synchrotrons
- X-ray free electron lasers
- Lab-based X-ray sources

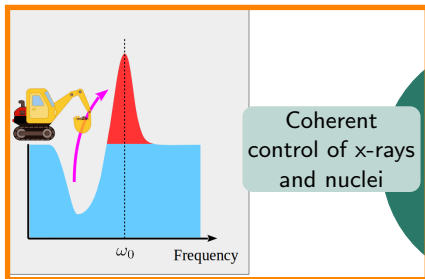
Control techniques

- Pulse shaping
- Coherent excitation control

- Shvyd'ko et. al. *Phys. Rev. Lett.* **77**, 3232 (1996)
 Bürvenich, Evers, Keitel *Phys. Rev. Lett.* **96** 142 (2006)
 Pálffy et. al. *Phys. Rev. Lett.* **103** 017401 (2009)
 Adams et. al. *J. Mod. Opt.* **60** 2 (2013)
 Vagizov et. al. *Nature* **508**, 80 (2014)
 Heeg et. al. *Science* **357** 6349 (2017)
 Heeg et. al. arXiv:1607.04116 [quant-ph]
 Goerttler et. al. *Phys. Rev. Lett.* **123**, 153902 (2019)

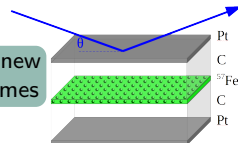
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Background & Motivation

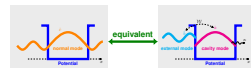


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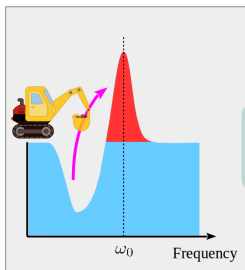
Control techniques

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Heeg, Kaldun, Strohm, Reiser, Ott, Subramanian, DL, Haber, Wille, Goerttler, Ruffer, Keitel, Röhlberger, Pfeifer, Evers, *Science* **357**, 375 (2017) + submitted

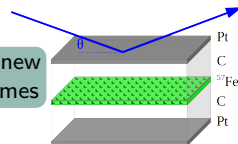
Background & Motivation



Coherent control of x-rays and nuclei

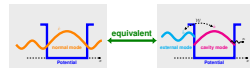
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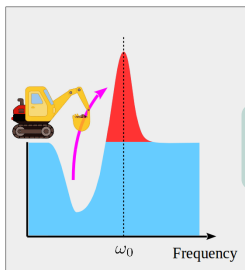


today

Understand fundamentals



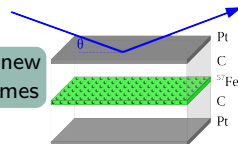
Background & Motivation



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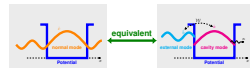
Nuclear quantum optics

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today

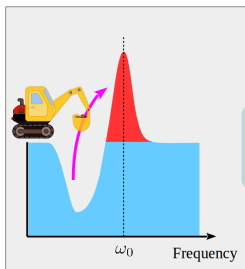
Understand fundamentals



Extreme regimes

⇒ New theoretical challenges

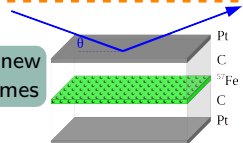
Background & Motivation



Coherent
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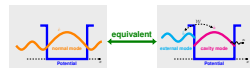
Nuclear
quantum
optics

X-rays excite new
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today

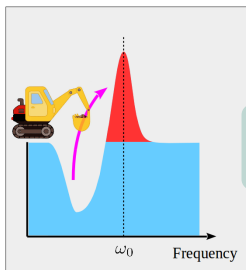
Understand
fundamentals



Extreme regimes

$\Rightarrow$ **New theoretical challenges**

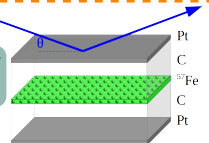
Background & Motivation



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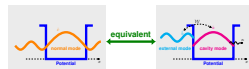
Nuclear
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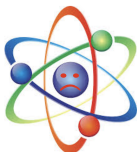
today

Understand
fundamentals



Extreme regimes

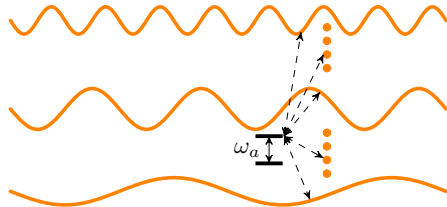
⇒ **New theoretical challenges**



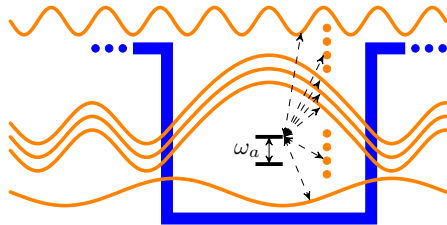
Extreme **atomic** systems

Overview

Continuum
coupling

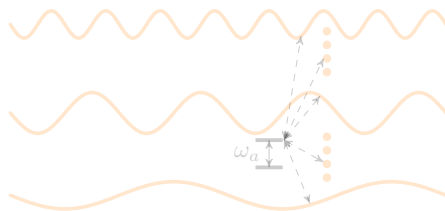


Structured continuum
featuring resonances



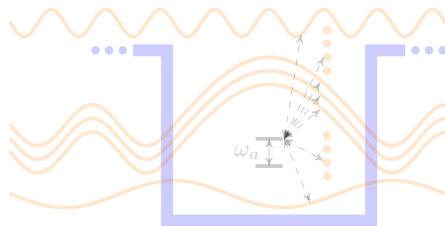
Overview

Continuum
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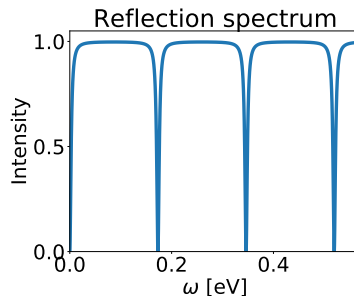
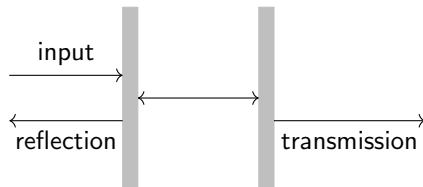


Can we extract **relevant** degrees of freedom from a continuum?

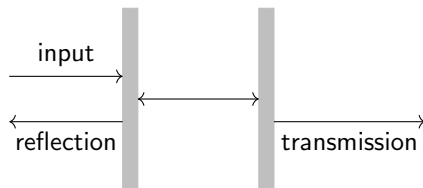
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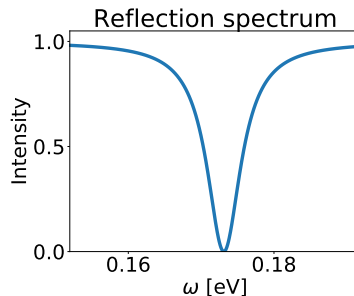
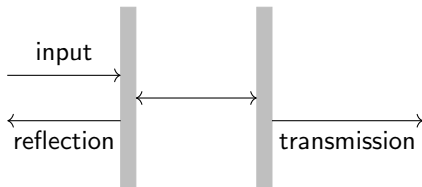
Example: Fabry-Perot cavity



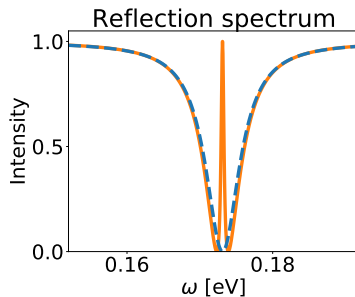
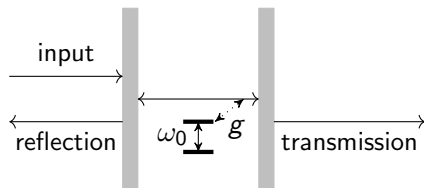
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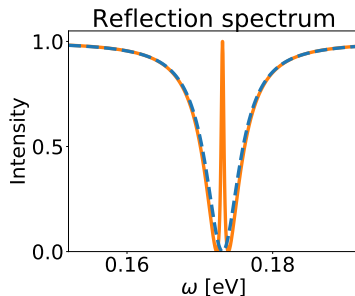
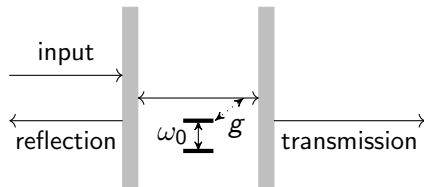
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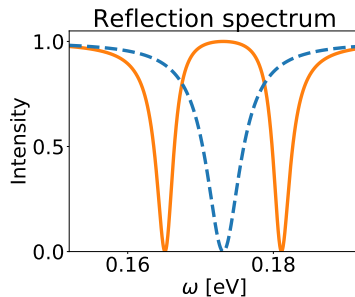
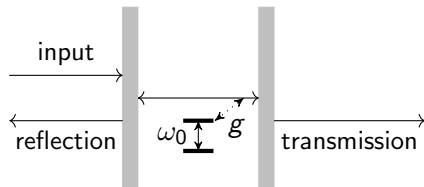


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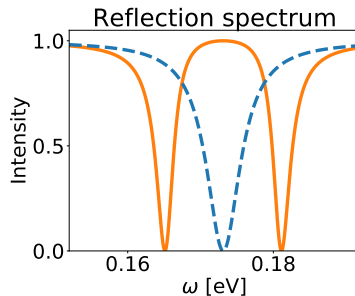
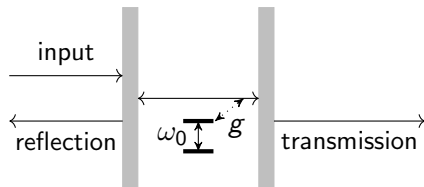
→ Weak coupling: Purcell effect

Example: Fabry-Perot cavity



→ Strong coupling: Vacuum Rabi-splitting

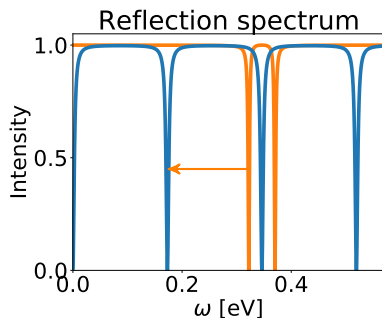
Example: Fabry-Perot cavity



⇒ Quantum effects via strong light-matter interactions!

Extreme regimes

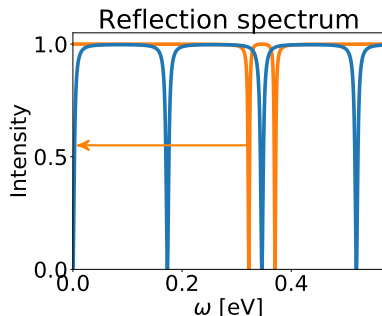
- Multi-mode strong coupling



Türeci et al. *Science* **320**, 643 (2008)
Krimmer et al. *Phys. Rev. A* **89**, 033820 (2014)
Sundaresan et al. *Phys. Rev. X* **5**, 021035 (2015)
... and many more ...

Extreme regimes

- Multi-mode strong coupling
- Ultra-strong coupling
- Deep-strong coupling

**Recent reviews:**

Carusotto & Ciuti *Rev. Mod. Phys.* **85**, 299 (2013)

Frisk Kockum et al. *Nat. Rev. Phys.* **1**, 19 (2019)

Forn-Díaz et al. *Rev. Mod. Phys.* **91**, 025005 (2019)

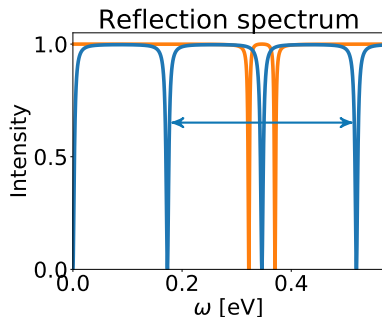
Experimental:

Niemczyk et al. *Nat. Phys.* **6**, 772 (2010)

...and many more ...

Extreme regimes

- Multi-mode strong coupling
- Ultra-strong coupling
- Deep-strong coupling
- Overlapping modes

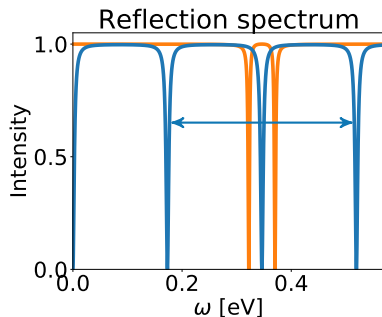


Petermann *IEEE J. Quantum Electron.* **15**, 566 (1979)
Hackenbroich, Viviescas & Haake *Phys. Rev. Lett.* **89**, 083902 (2002)
I. Rotter *J. Phys. A: Mathematical and Theoretical* **45**, 15 (2009)
Heeg et al. *Phys. Rev. Lett.* **114**, 207401 (2015)
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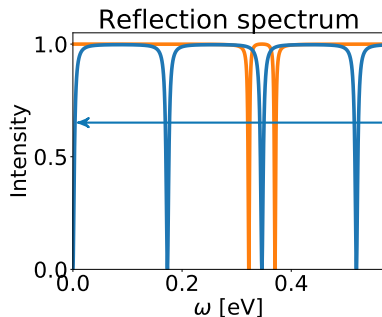
e.g. non-Hermitian
photonics



Petermann *IEEE J. Quantum Electron.* **15**, 566 (1979)
Hackenbroich, Viviescas & Haake *Phys. Rev. Lett.* **89**, 083902 (2002)
I. Rotter *J. Phys. A: Mathematical and Theoretical* **45**, 15 (2009)
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Extreme regimes

- Multi-mode strong coupling
- Ultra-strong coupling
- Deep-strong coupling
- Overlapping modes
- Large leakage

**Experimentally relevant:**

Altewischer et al. *Nature* **418**, 304306 (2002)

Savage et al. *Nature* **491**, 574577 (2012)

Esteban et al. *Nat. Comm.* **3**, 825 (2012)

Tame et al. *Nat. Phys.* **9**, 329340 (2013)

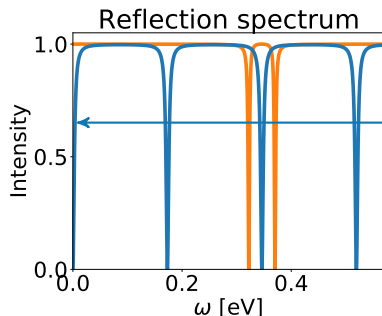
...and many more ...

Extreme regimes

Extreme coupling

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- Large leakage

Extreme openness



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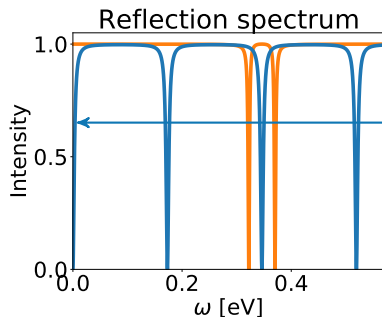
Extreme regimes

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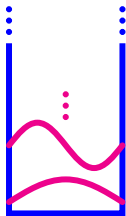
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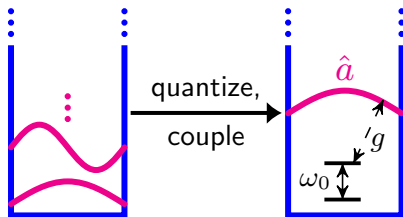
Extreme complexity (e.g. many-body)



From closed to open boxes



From closed to open boxes

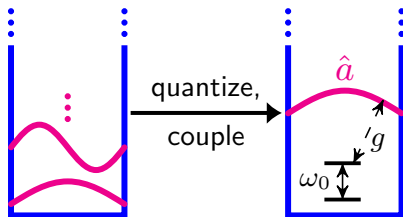


Jaynes-Cummings & friends

$$H = H_{\text{atom}} + H_{\text{cav}} + g\hat{a}\hat{\sigma}^+ + h.c.$$

→ **few-mode concept**

From closed to open boxes



- discrete eigenstates
- closed system
 - × no leakage
 - × no driving
 - × no scattering
 - × no external detection

Jaynes-Cummings & friends

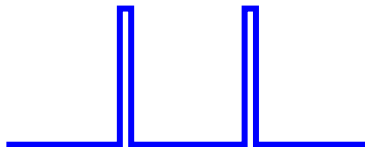
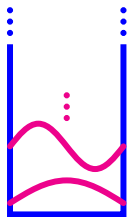
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→ **few-mode concept**

From closed to open boxes

open system

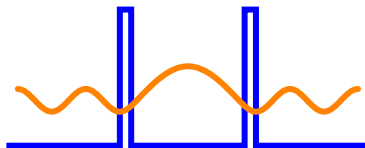
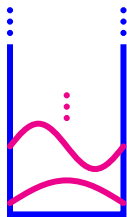
- ✓ leakage
- ✓ driving
- ✓ scattering



From closed to open boxes

open system

- ✓ leakage
- ✓ driving
- ✓ scattering

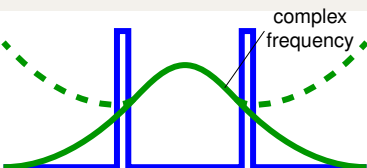


continuum eigenstates

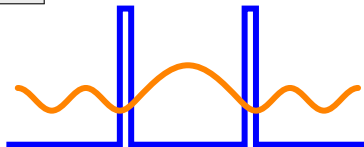


few-mode concept
is lost

From closed to open boxes



resonant modes

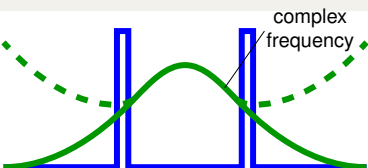


continuum eigenstates

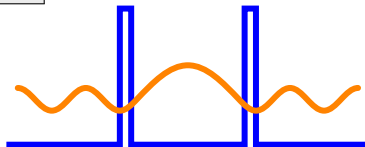


few-mode concept
is lost

From closed to open boxes



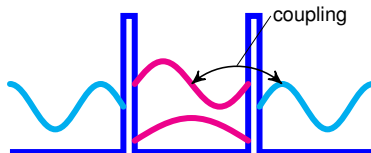
resonant modes



continuum eigenstates

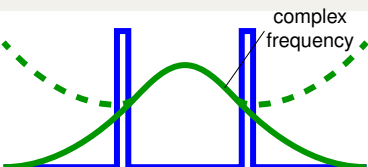


few-mode concept
is lost



few-mode + input-output

From closed to open boxes

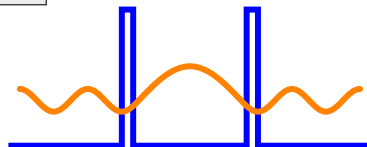


resonant modes

$$\underline{\underline{G}}(\mathbf{r}, \mathbf{r}', \omega)$$

$$n(\mathbf{r}, \omega) \quad \rho(\mathbf{r}, \omega)$$

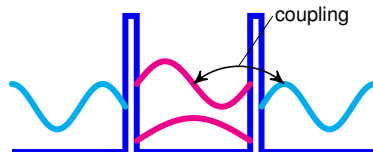
Green fns, LDOS, Master eqs.



continuum eigenstates

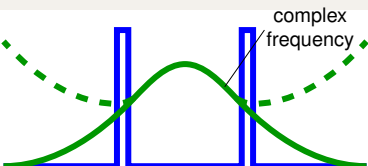


few-mode concept
is lost



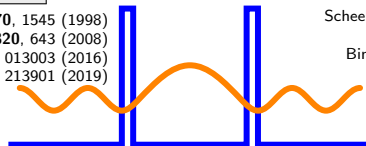
few-mode + input-output

From closed to open boxes



resonant modes

Ching et al. *Rev. Mod. Phys.* **70**, 1545 (1998)
 Türeci et al. *Science* **320**, 643 (2008)
 Cerjan & Stone *Phys. Scr.* **91** 013003 (2016)
 Franke et al. *PRL* **122**, 213901 (2019)



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$$n(\mathbf{r}, \omega) \quad \rho(\mathbf{r}, \omega)$$

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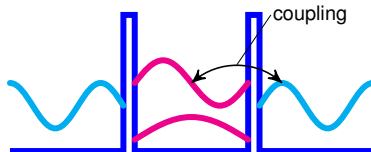
Scheel & Buhmann *Acta Phys. Slov.* **58**, 675 (2008)
 Krimer et al. *Phys. Rev. A* **89**, 033820 (2014)
 Binninger et al. *Phys. Rev. A* **100**, 033816 (2019)

continuum eigenstates



few-mode concept
is lost

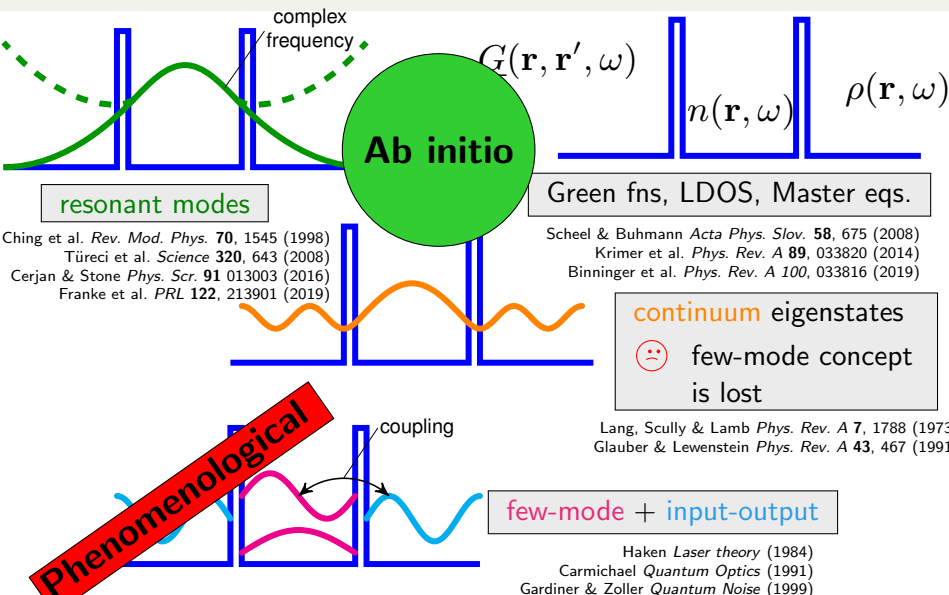
Lang, Scully & Lamb *Phys. Rev. A* **7**, 1788 (1973)
 Glauber & Lewenstein *Phys. Rev. A* **43**, 467 (1991)



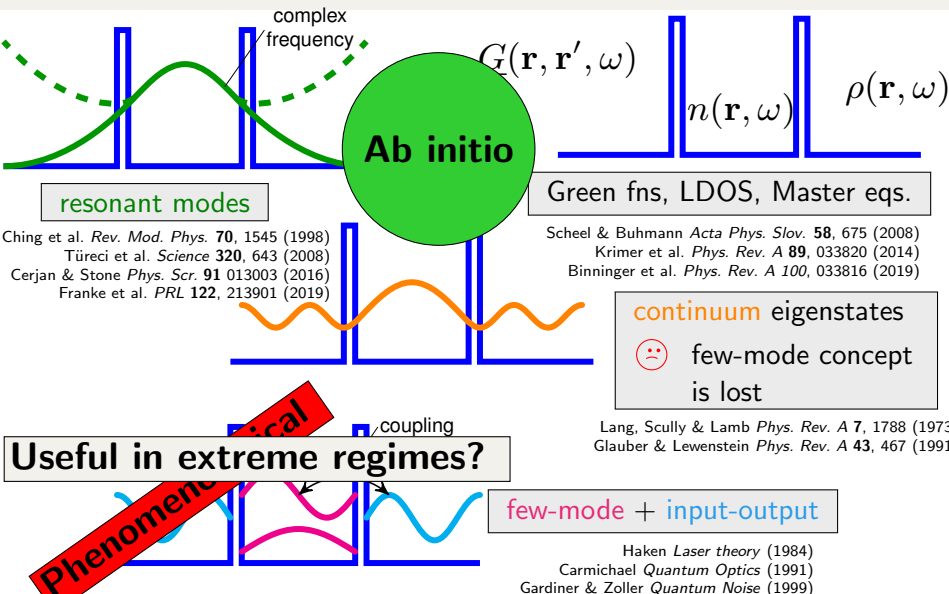
few-mode + input-output

Haken *Laser theory* (1984)
 Carmichael *Quantum Optics* (1991)
 Gardiner & Zoller *Quantum Noise* (1999)

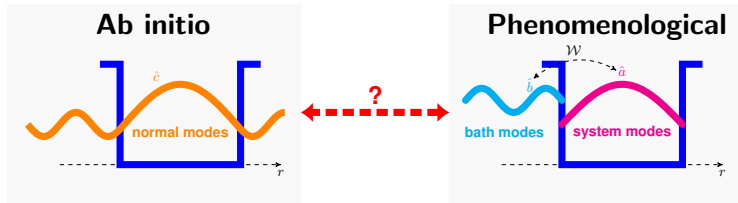
From closed to open boxes



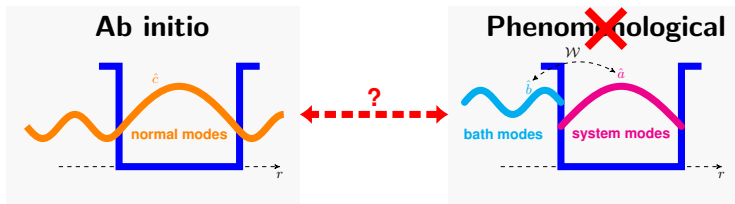
From closed to open boxes



The problem



The problem



How to make

- **few-mode**

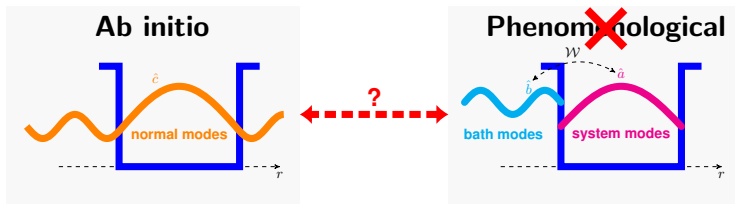
and

- **input-output**

ab initio?



The problem



How to make

- **few-mode**

and

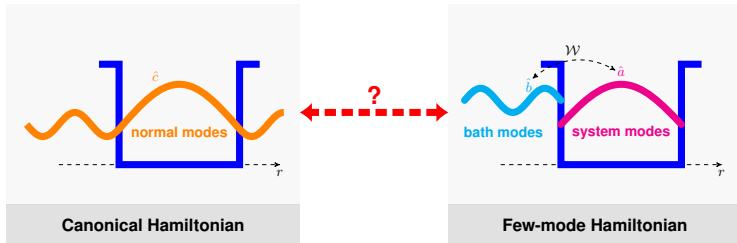
- **input-output**

ab initio?

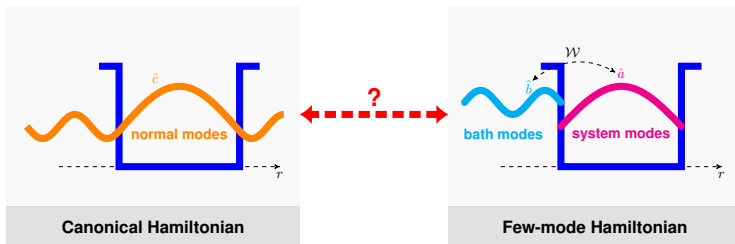


⇒ **Ab initio few-mode theory**

Ab initio few-mode Hamiltonians



Ab initio few-mode Hamiltonians



normal modes

$$\hat{c}(\omega)$$

discrete modes

$$= \sum_{\lambda} \alpha_{\lambda}(\omega) \hat{a}_{\lambda}$$

external continuum

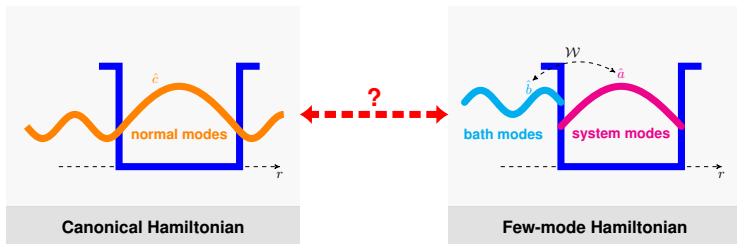
$$+ \int d\omega' \beta(\omega, \omega') \hat{b}(\omega')^{1,2}$$

¹Viviescas & Hackenbroich, *Phys. Rev. A* **67**, 013805 (2003)

²Domcke, *Phys. Rev. A* **28**, 2777 (1982)

³DL & J. Evers, *Phys. Rev. X* **10**, 011008 (2020)

Ab initio few-mode Hamiltonians



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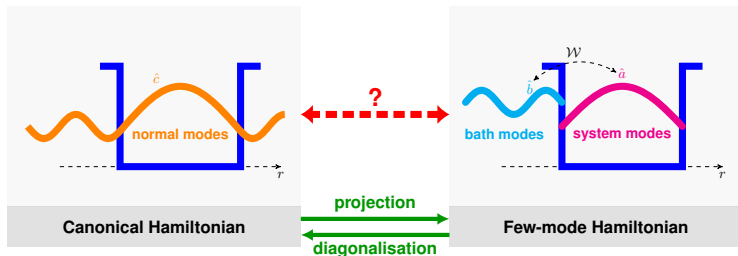
$\Rightarrow$ select **resonant states** as few-mode basis³

¹Viviescas & Hackenbroich, *Phys. Rev. A* **67**, 013805 (2003)

²Domcke, *Phys. Rev. A* **28**, 2777 (1982)

³DL & J. Evers, *Phys. Rev. X* **10**, 011008 (2020)

Ab initio few-mode Hamiltonians



$$\text{normal modes} \quad \hat{c}(\omega) = \sum_{\lambda} \alpha_{\lambda}(\omega) \hat{a}_{\lambda} + \int d\omega' \beta(\omega, \omega') \hat{b}(\omega')^{1,2}$$

$\hat{c}(\omega)$
 $\sum_{\lambda} \alpha_{\lambda}(\omega) \hat{a}_{\lambda}$
 $+ \int d\omega' \beta(\omega, \omega') \hat{b}(\omega')$

⇒ select **resonant states** as few-mode basis³

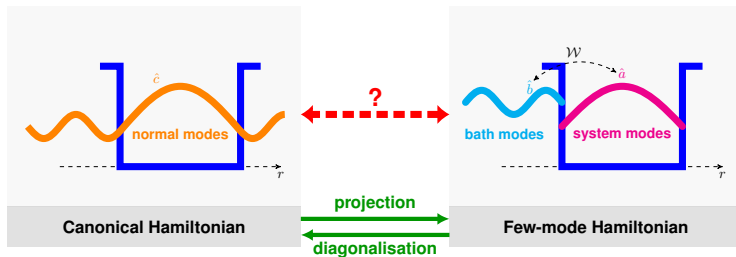
⇒ **ab initio few-mode Hamiltonians** 😊³

¹Viviescas & Hackenbroich, *Phys. Rev. A* **67**, 013805 (2003)

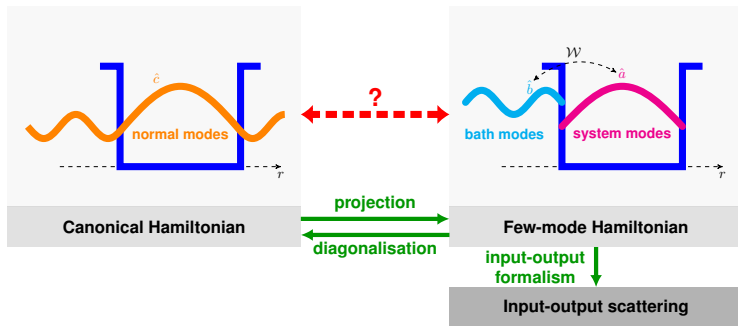
²Domcke, *Phys. Rev. A* **28**, 2777 (1982)

³DL & J. Evers, *Phys. Rev. X* **10**, 011008 (2020)

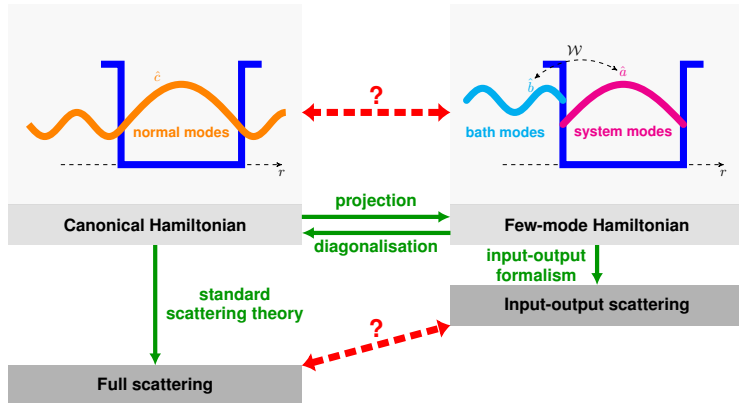
Few-mode scattering



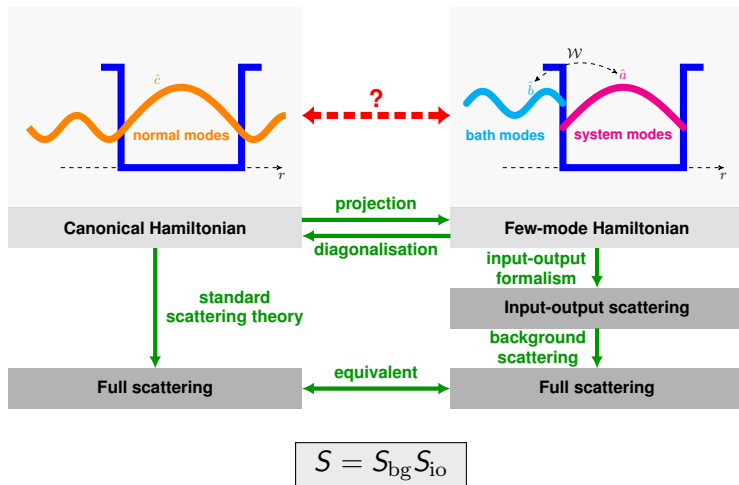
Few-mode scattering



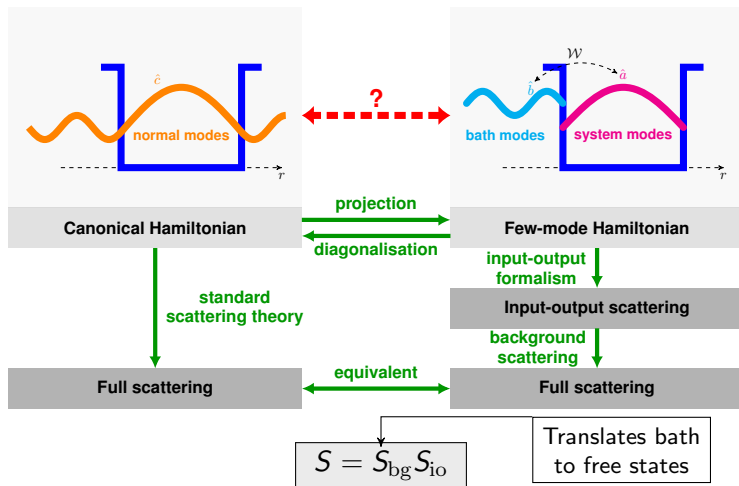
Few-mode scattering



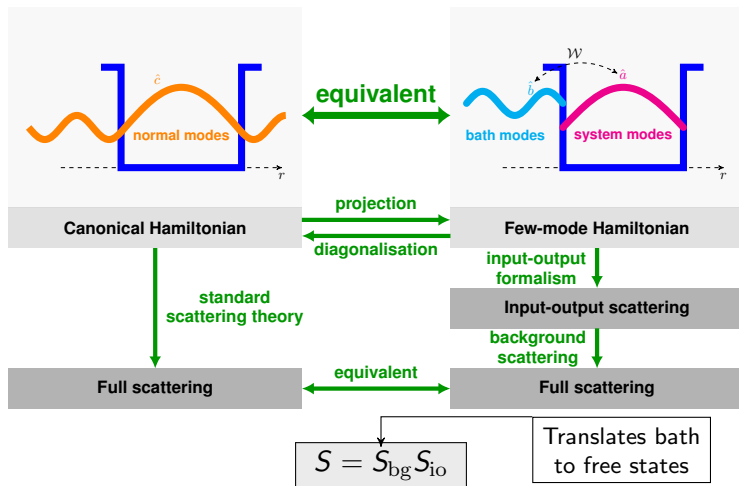
Few-mode scattering



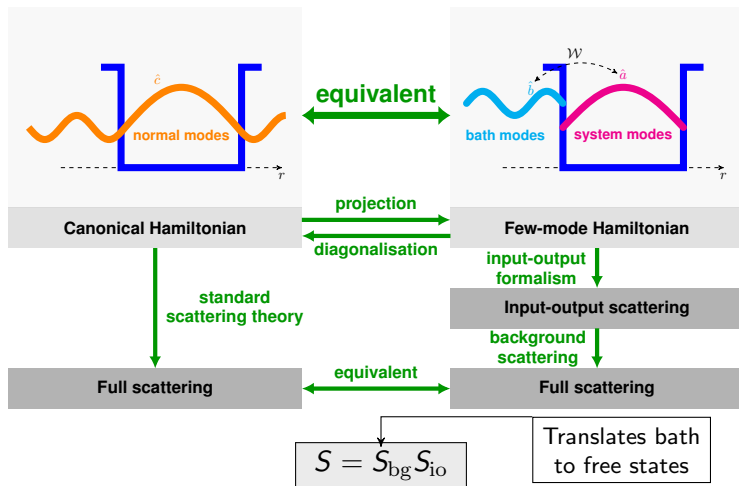
Few-mode scattering



Few-mode scattering



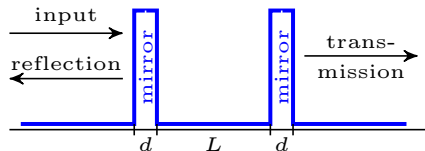
Few-mode scattering



$\Rightarrow$ Few-mode theory can apply in extreme regimes!

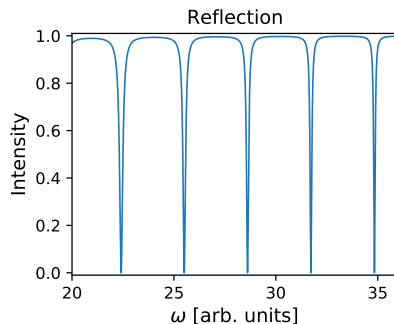
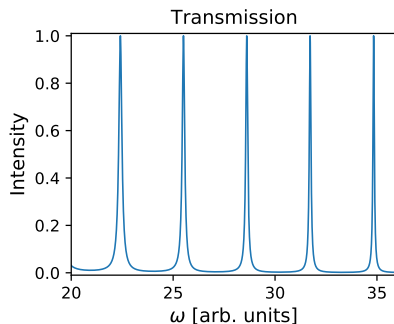
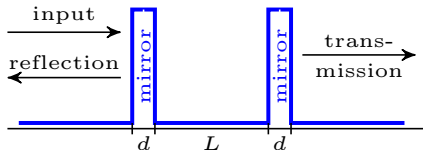
Illustrative example

Example: Two-sided Fabry-Perot cavity

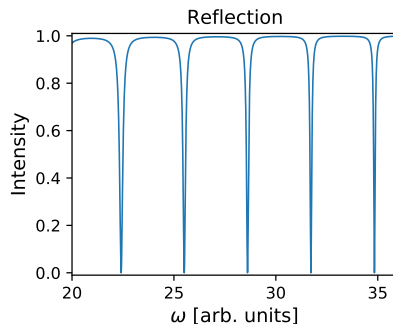
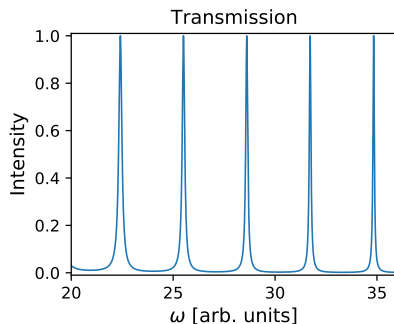
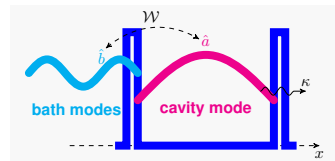
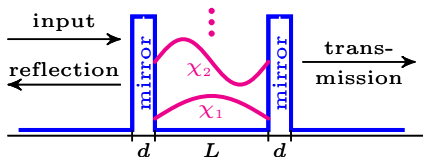


Ley & Loudon *J. Mod. Opt.* **34**, 227-255 (1987)

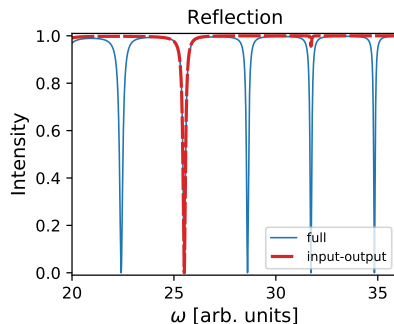
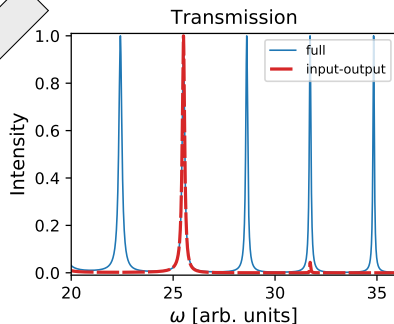
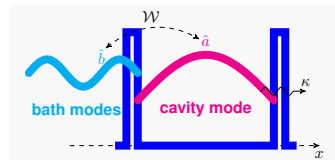
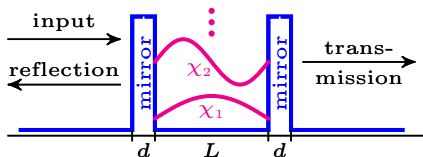
Example: Two-sided Fabry-Perot cavity



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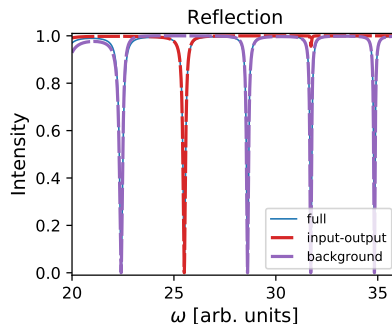
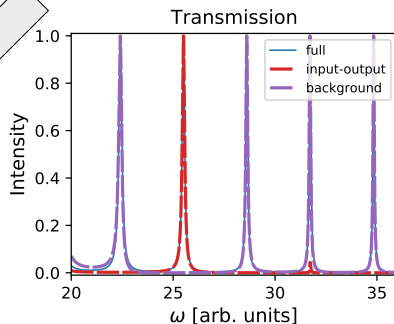
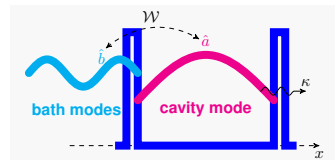
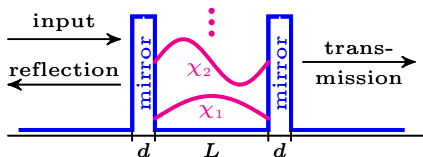


Example: Two-sided Fabry-Perot cavity



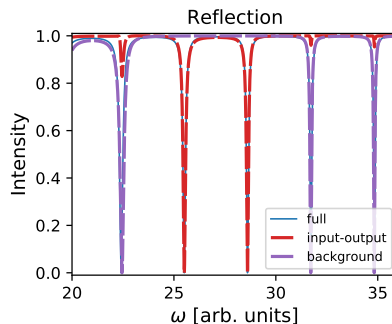
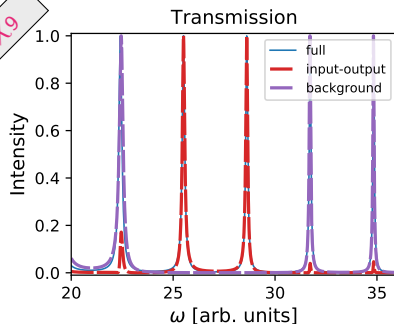
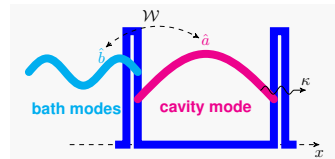
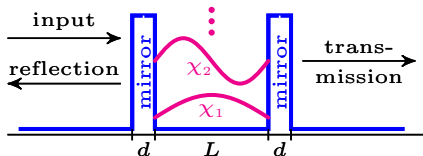
$\Rightarrow$ ab initio, not a fit!

Example: Two-sided Fabry-Perot cavity



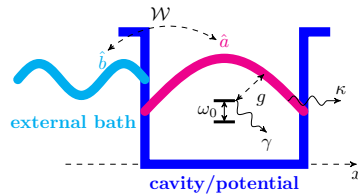
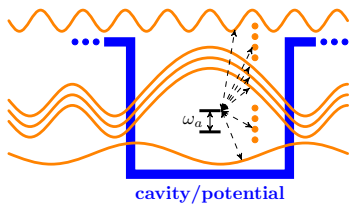
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Example: Two-sided Fabry-Perot cavity

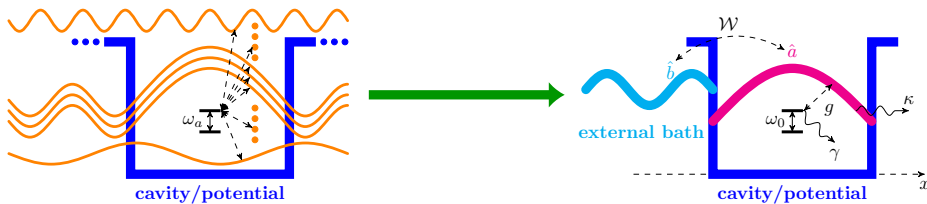


⇒ extract resonant dynamics

Interacting systems

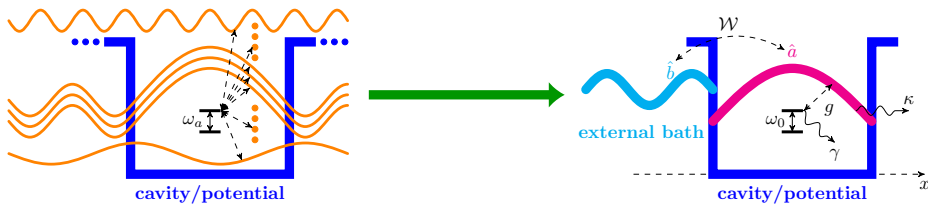


Interacting systems



- Many degrees of freedom
- Often difficult!

Interacting systems



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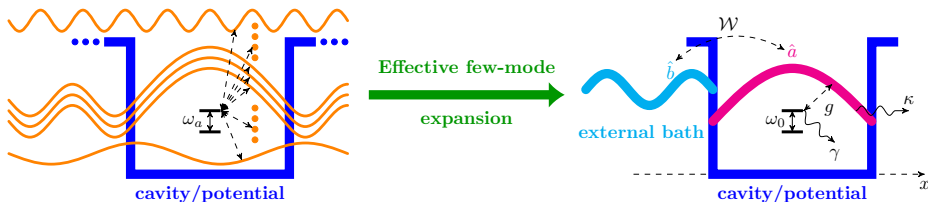
- Much easier to solve!
 - Many methods already exist!^{1,2,3}
 - Advantages to model version
- ⇒ Non-interacting system exact!

¹ Carmichael, *Statistical Methods in Quantum Optics 1* (1999)

² Gardiner & Zoller *Quantum Noise* (1999)

³ Kirton et al. *Adv. Quantum Technol.* **2**, 1800043 (2019)

Interacting systems



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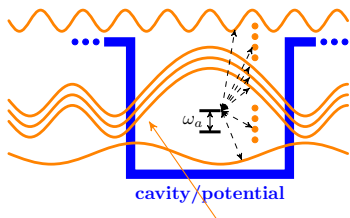
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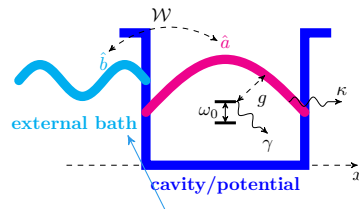
² Gardiner & Zoller *Quantum Noise* (1999)

³ Kirton et al. *Adv. Quantum Technol.* **2**, 1800043 (2019)

Interacting systems



Effective few-mode
expansion



non-perturbative/non-Markovian

perturbative/Markovian

- Many degrees of freedom
- Often difficult!

- Much easier to solve!
 - Many methods already exist!^{1,2,3}
 - Advantages to model version
- ⇒ Non-interacting system exact!

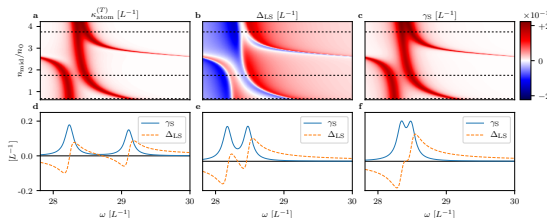
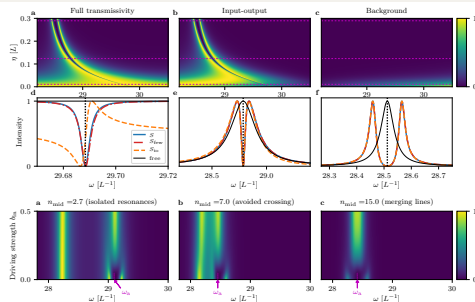
¹ Carmichael, *Statistical Methods in Quantum Optics I* (1999)

² Gardiner & Zoller *Quantum Noise* (1999)

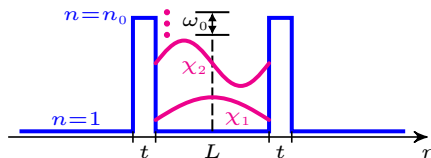
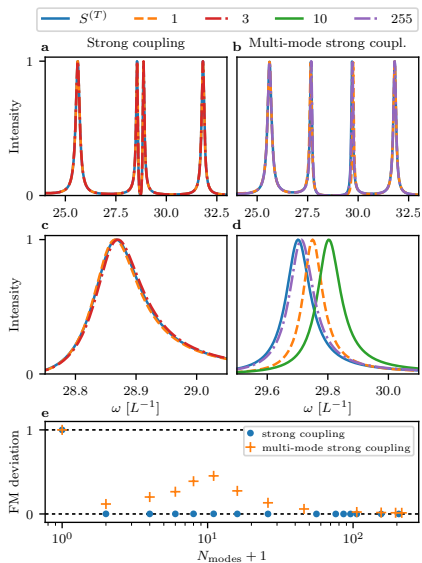
³ Kirton et al. *Adv. Quantum Technol.* **2**, 1800043 (2019)

Benchmarks and advantages

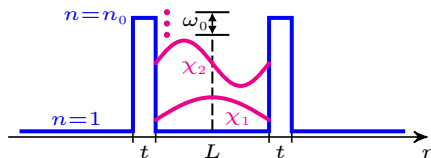
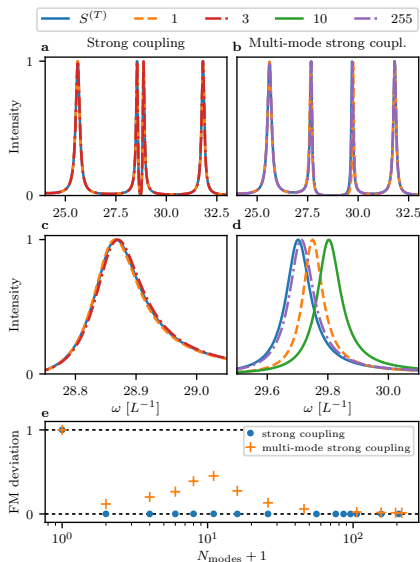
- ✓ Benchmarked in linear regime
- ✓ Highly open systems
- ✓ Non-linear effects
- ✓ Overlapping modes features
⇒ Non-trivial bath effects!
- ✓ Ab initio calculation of quantum couplings



Convergence and extreme regimes



Convergence and extreme regimes

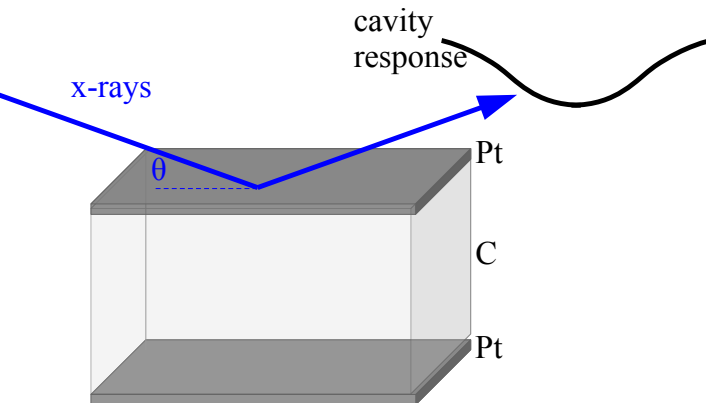


Convergence of light-matter coupling models is non-trivial!¹

¹ e.g. Krimer et al. *Phys. Rev. A* **89**, 033820 (2014)
 Malekakhlagh, Petrescu, Türeci *Phys. Rev. Lett.* **119**, 073601 (2017)
 Gely et al. *Phys. Rev. B* **95**, 245115 (2017)

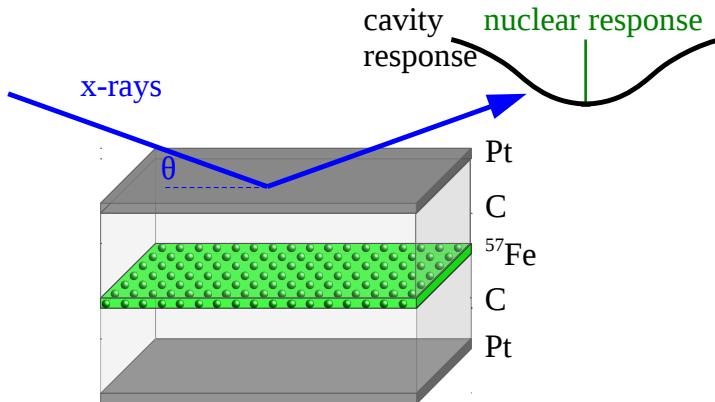
X-ray cavity QED

New experimental platform:



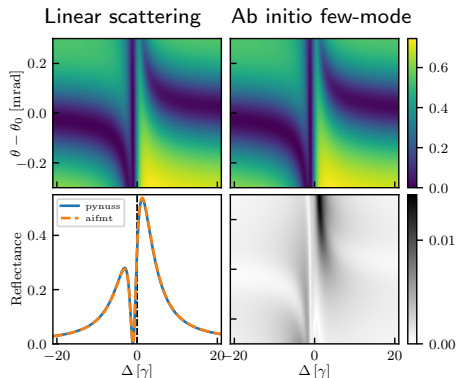
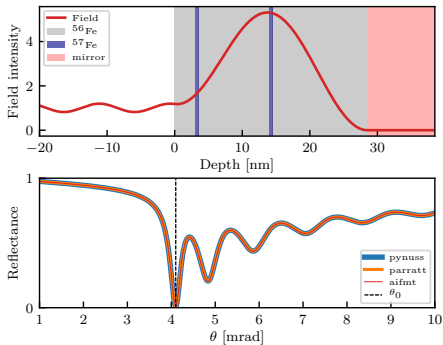
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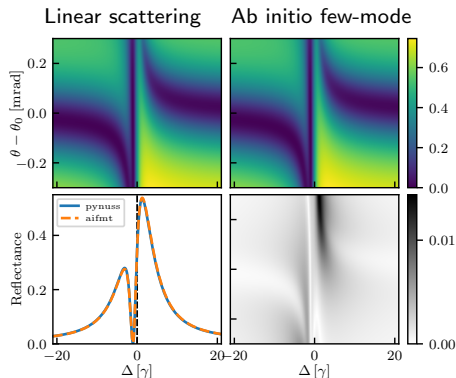
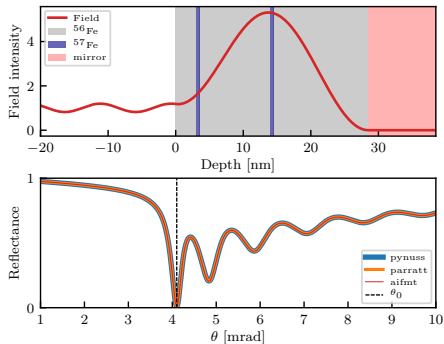
Fabry-Perot + Many-body ensemble

Ab initio few-mode theory for X-ray cavity QED



DL, Heeg, Keitel & Evers, *to be submitted*

Ab initio few-mode theory for X-ray cavity QED



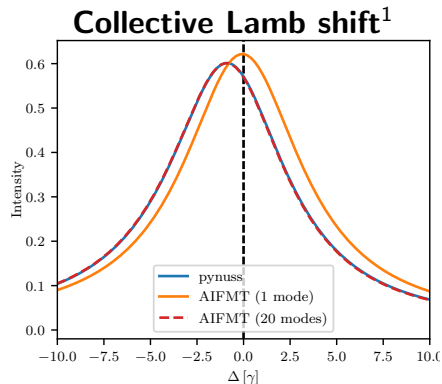
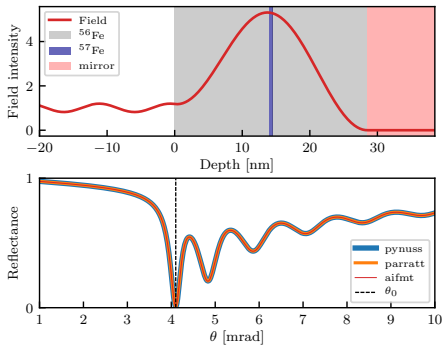
DL, Heeg, Keitel & Evers, *to be submitted*

$\Rightarrow$ improves successful phenomenological model^{1,2}

¹Heeg & Evers *PRA* (2013)

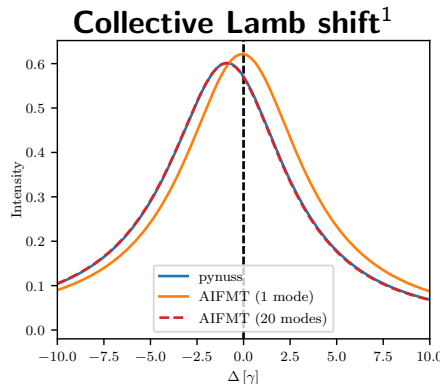
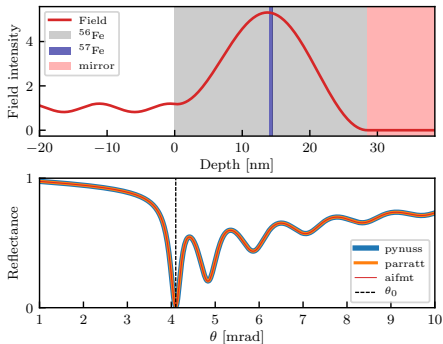
²Heeg & Evers *PRA* (2015)

Overlapping modes effects beyond input-output models



¹Röhlberger et. al. *Science* **328** 5983 (2010)

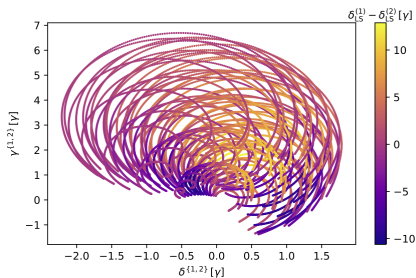
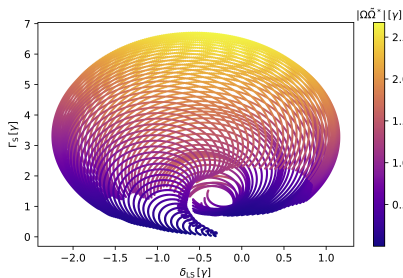
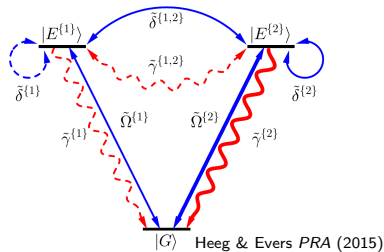
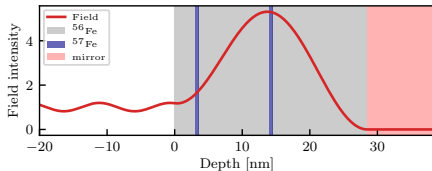
Overlapping modes effects beyond input-output models



¹Röhlberger et. al. *Science* **328** 5983 (2010)

$\Rightarrow$ **overlapping modes effect**

Towards designing nuclear level schemes



Conclusion

- ✓ Rigorous construction of few-mode Hamiltonians
- ✓ Exact scattering theory via input-output formalism
- ✓ Non-perturbative expansion scheme for interactions
- ✓ Linking ab initio theory and models in cavity QED
 - ⇒ Access to new regimes!
- !! Explore quantum effects in X-ray cavities
 - ⇒ In progress: non-linear and correlation effects at XFELs
- ?! Extreme regimes of open quantum dynamics

Thank you for your attention!



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