MAX-PLANCK-INSTITUT FÜR KERNPHYSIK HEIDELBERG

Ab initio few-mode theory

for quantum potential scattering problems

Dominik Lentrodt*, K. P. Heeg, C. H. Keitel & J. Evers

Non-interacting theory: Equivalence and resonance extraction

Quantum potential scattering

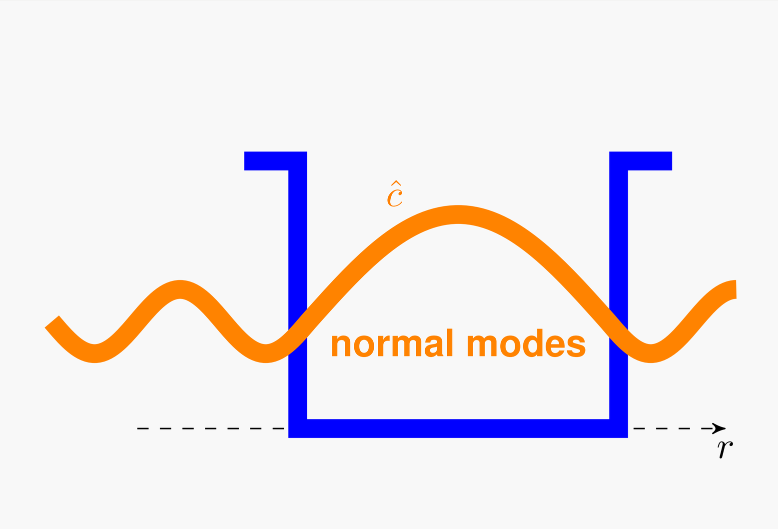
Wave equations with a potential [1, 2]:

$i\dot{\psi} = [-\nabla^2 + V(r)]\psi$ — Schrödinger equation

$\varepsilon(r)\ddot{\mathbf{A}} = -\nabla \times \nabla \times \mathbf{A}$ — Maxwell wave equation

⇒ Quantized Hamiltonian: $\hat{H}_{\text{cav}} = \int dk\omega(k)\hat{c}^{\dagger}(k)\hat{c}(k)$

⇒ With interactions: **hard problem**



Ab initio few-mode theory

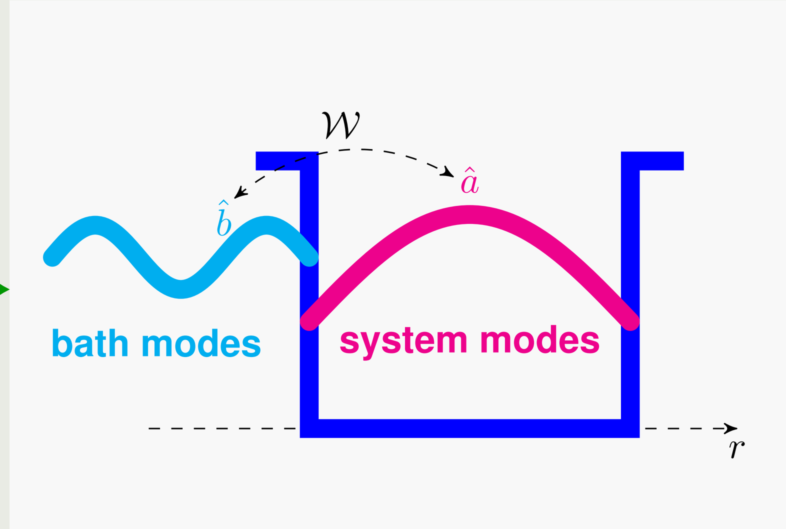
System-bath $\hat{H}_{\text{cav}}$ **equivalent** to diagonal scattering

$$\hat{H}_{\text{cav}} = \sum_{\lambda} \hbar\omega_{\lambda}\hat{a}^{\dagger}_{\lambda}\hat{a}_{\lambda} + \int dk\hbar\omega_{k}\hat{b}^{\dagger}_{k}\hat{b}_{k} + \int dk \sum_{\lambda} \mathcal{W}_{\lambda k}\hat{a}^{\dagger}_{\lambda}\hat{b}_{k} + h.c.$$

⇒ **Gardiner-Collett Hamiltonian** [3, 4]

⇒ Discrete set of system modes

⇒ **Solvable** via input-output formalism [3, 4]



equivalent

Canonical Hamiltonian — projection → Few-mode Hamiltonian

diagonalization ←

Canonical Hamiltonian ↓ standard scattering theory ↓ Full scattering

Few-mode Hamiltonian ↓ input-output formalism ↓ Input-output scattering ↓ background scattering ↓ Full scattering

Full scattering — equivalent — Full scattering

Lentrodt & Evers, *Phys. Rev. X* **10**, 011008 (2020)

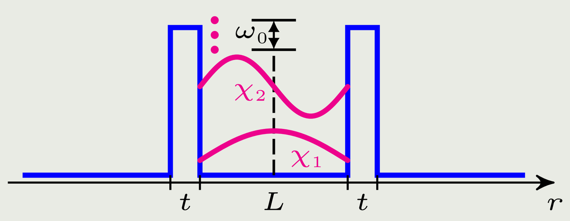
The formalism

Feshbach projections [5, 6] gives system-bath expansion of normal mode operators

$$\hat{c}(k) = \sum_{\lambda} \alpha_{\lambda}(k)\hat{a}_{\lambda} + \int dk'\beta(k,k')\hat{b}_{k}$$

⇒ Choose states to **extract resonances**
- Simplify continuum structure
- Relevant dynamical degrees of freedom captured

Illustration of the concept



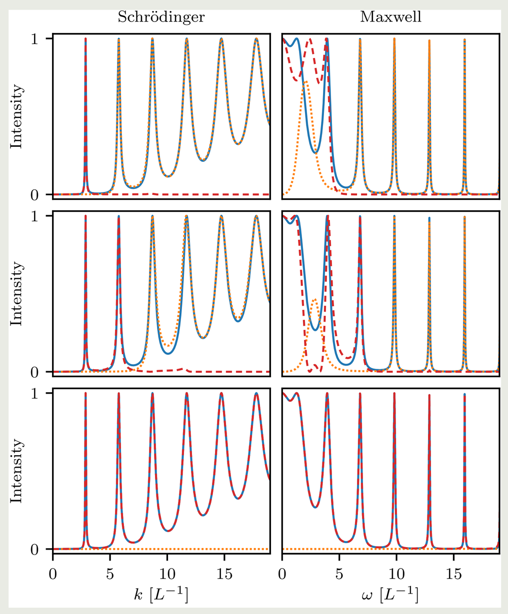
- Resonance selection without approximation
- Example: Fabry-Perot

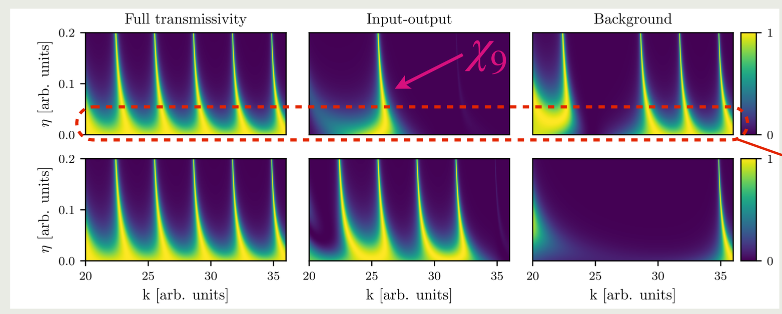
Chosen few-mode basis

χ_1 only

$\chi_1 + \chi_2$

$\chi_1, \cdots, \chi_{100}$



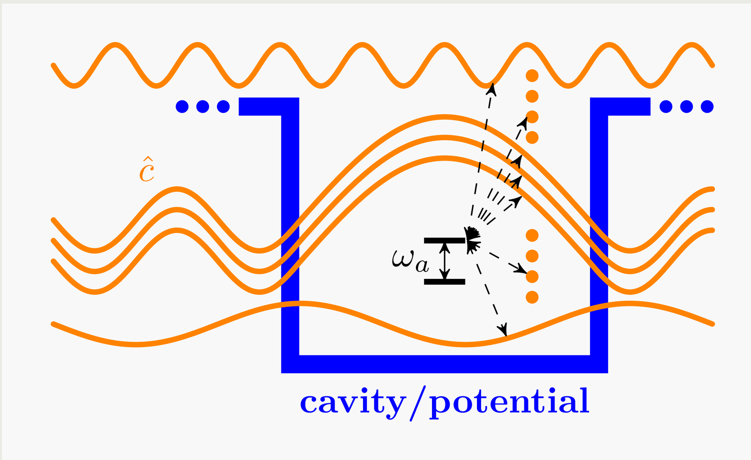


Overlapping modes regime!

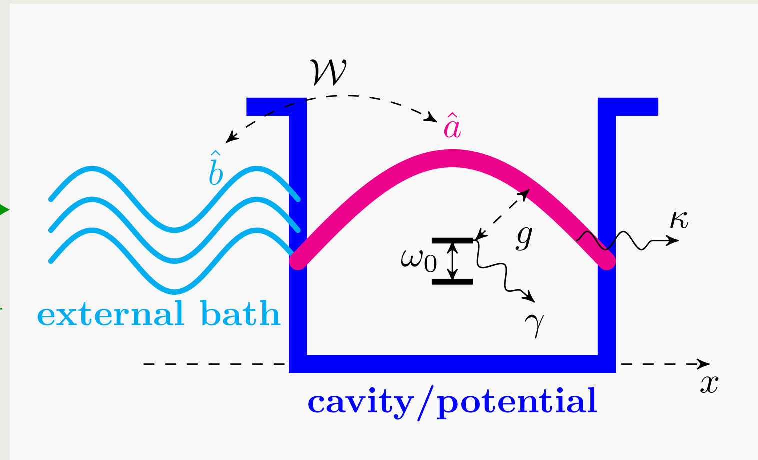
Interactions: Few-mode expansions

Effective few-mode expansions

- **Extract** relevant degrees of freedom from a continuum in an open quantum system + reduce Hilbert space size
- **Simplifies** the theoretical pictures ⇒ Many methods already exist in few-mode theory!
- Our ab initio theory provides **practical advantages** to phenomenological models!



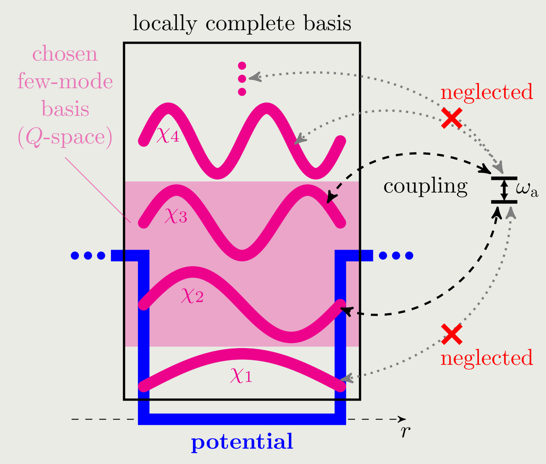
Effective few-mode expansion

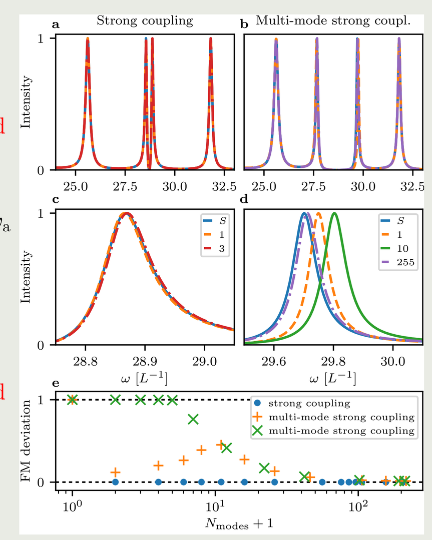


Interfaces with recent developments [7, 8, 9]

Multi-mode convergence

Advantages:
- Non-interacting system treated exactly!
- Systematic expansion disentangles approximations
- Direct connection to existing toolbox
- Access to extreme regimes

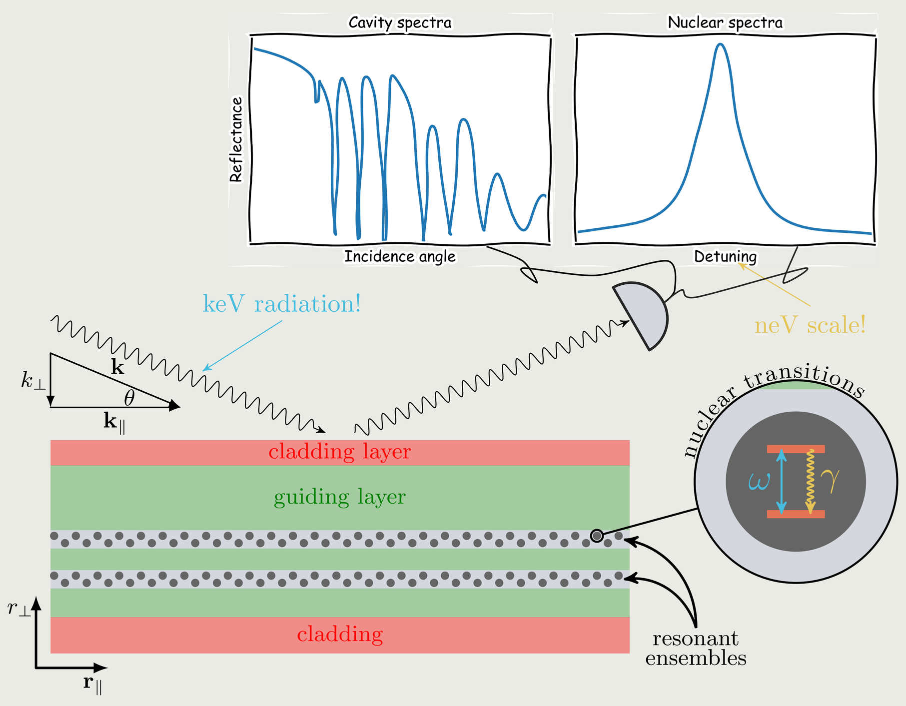


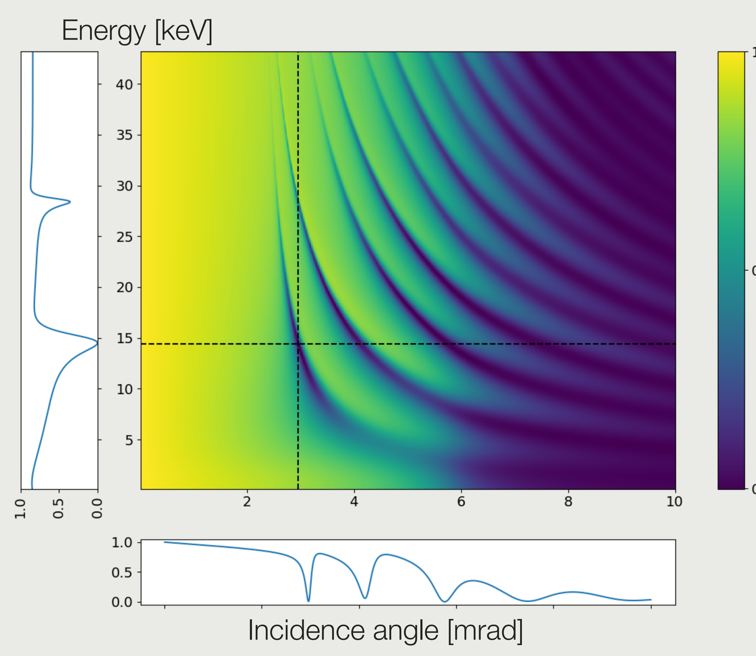


Convergence of light-matter coupling models is non-trivial [e.g. 10]

Hard X-ray cavity QED with Mössbauer nuclei

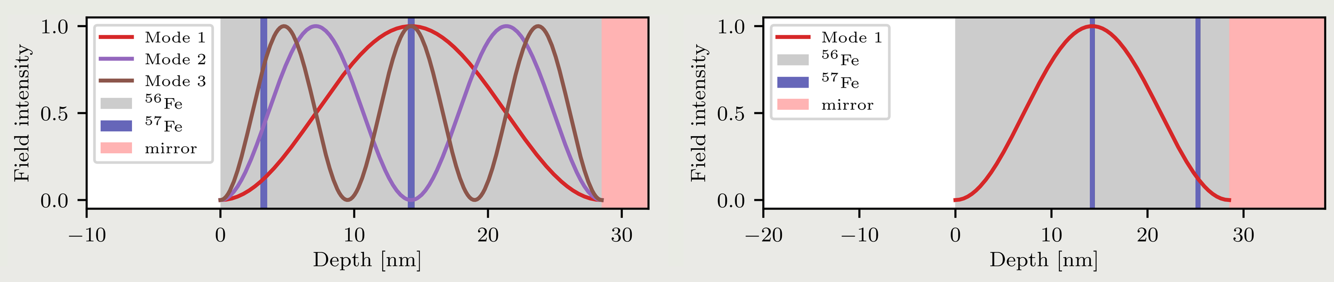
Many-body cavity QED at the energy frontier [11, 12, 13, ...]





Mode contributions in control-field-free EIT [14]

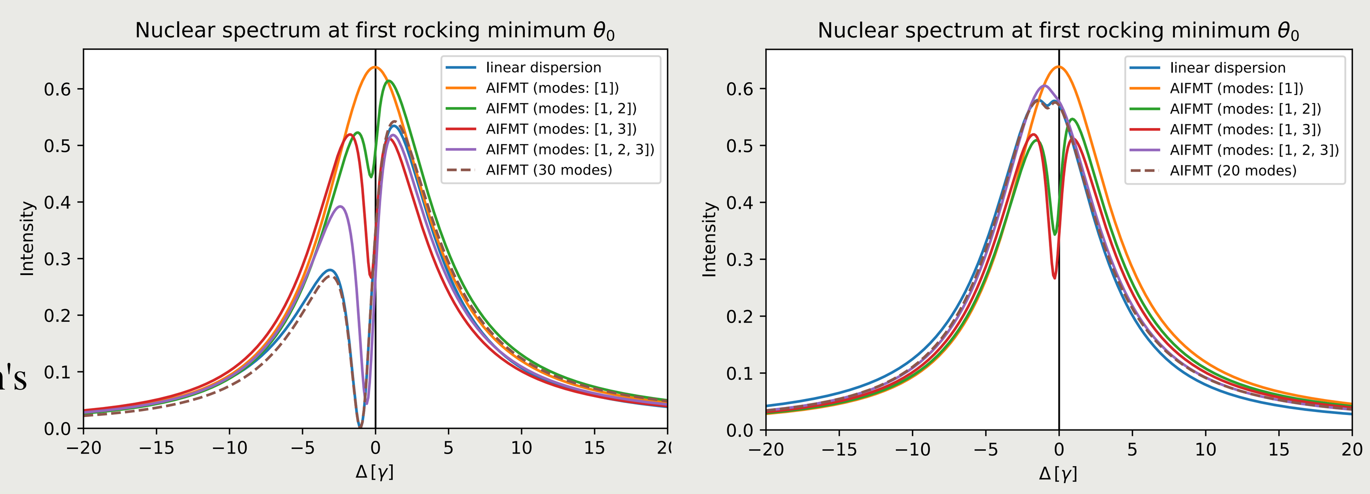
- First observed in [15]
- EIT without a control field
- Two similar configurations:
  - ⇒ Only one shows EIT!
  - ⇒ Why?



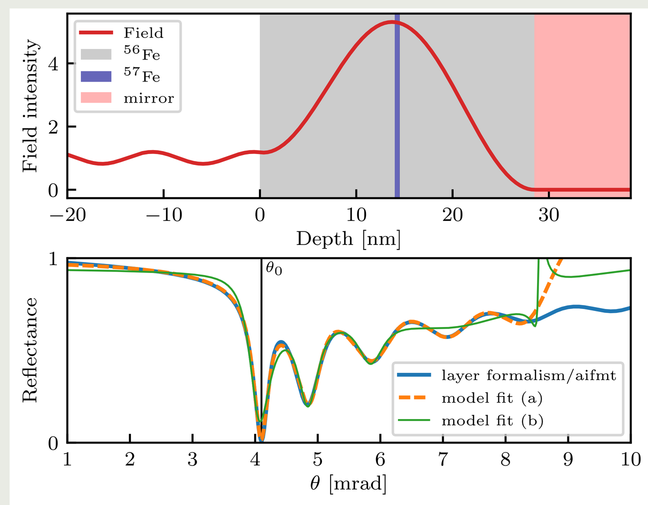
⇒ **Phase sensitive mode coupling**

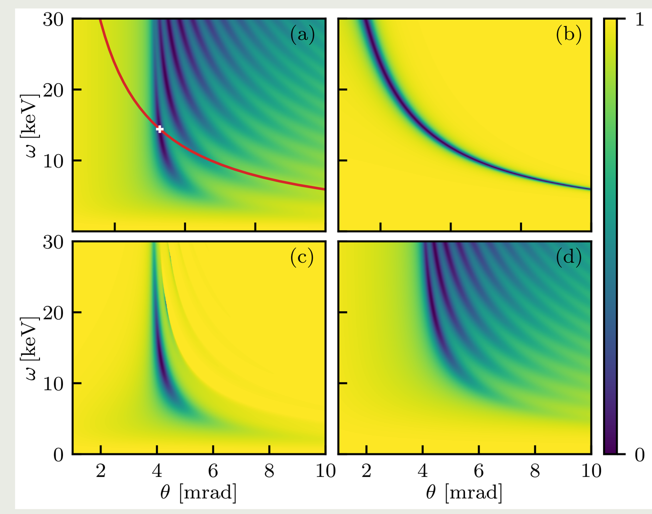
⇒ Towards designing effective nuclear level schemes

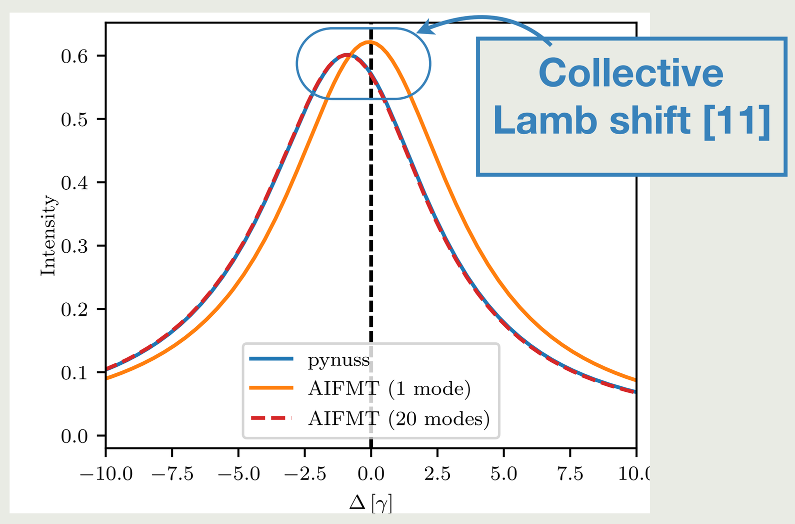
⇒ More insights from Green's function methods [17, 18]

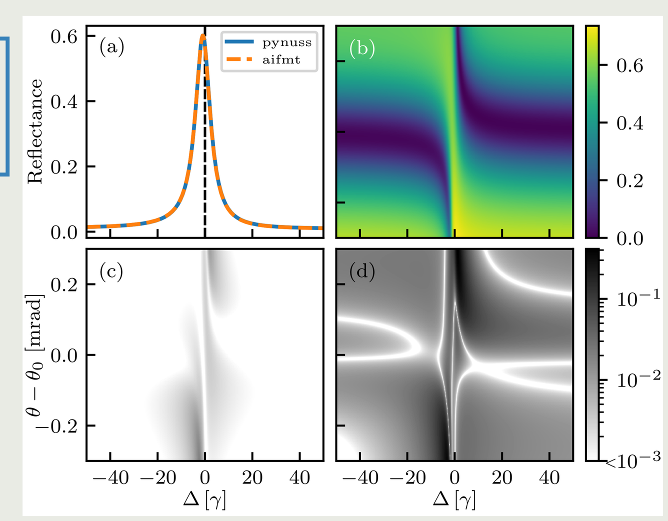


Beyond input-output models [6]

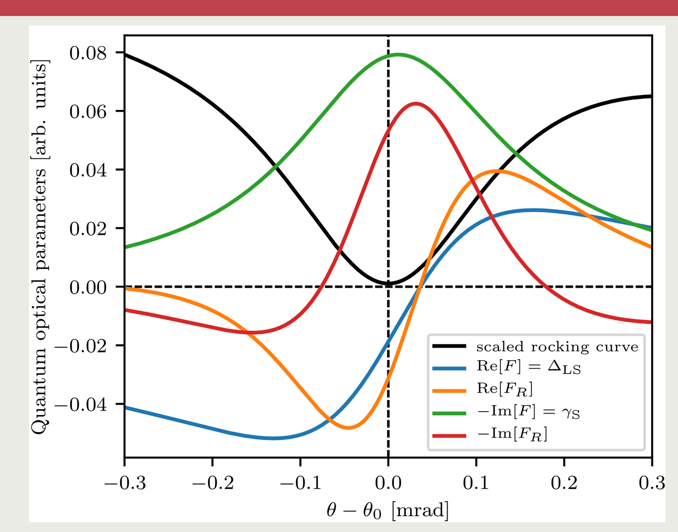


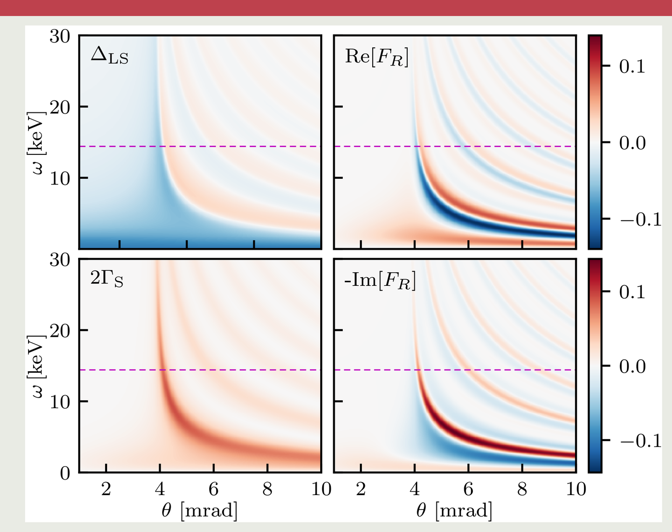






Ab initio quantum parameters [16]





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***Contact:**
Dominik Lentrodt
dominik.lentrodt@mpi-hd.mpg.de
MPI für Kernphysik, Heidelberg
PhD supervisor: Prof. Dr. Jörg Evers
Division: Prof. Dr. Christoph H. Keitel